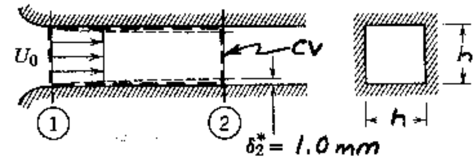
Problem 9.18

Given: Flow in the entrance region of a square duct as shown.

$U_0 = 30\,m/s$



Fluid is air.

$h = 80\,mm$

Find: Pressure change between sections ① and ②.

Solution: Apply continuity equation to find V_2, then use Bernoulli equation to find pressure change.

Basic equations: $$0 = \overset{=0(1)}{\frac{\partial}{\partial t}\int_{CV} \rho\, d\forall} + \int_{CS} \rho \vec{V}\cdot d\vec{A}$$

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + \overset{(5)}{g z_1} = \frac{p_2}{\rho} + \frac{V_2^2}{2} + \overset{(5)}{g z_2}$$

Assumptions: (1) Steady flow
(2) Incompressible flow
(3) No friction outside boundary layers
(4) Flow along a streamline
(5) $z_1 = z_2$

Then

$$0 = \{-|\rho V_1 A_1|\} + \{|\rho V_2 A_2|\} \quad \text{or} \quad V_1 A_1 = V_2 A_2 \quad \text{or} \quad V_2 = V_1 \frac{A_1}{A_{2,eff}}$$

and $$p_1 - p_2 = \frac{\rho}{2}(V_2^2 - V_1^2) = \frac{\rho V_1^2}{2}\left[\left(\frac{A_1}{A_{2,eff}}\right)^2 - 1\right]$$

At section ①, the area is $A_1 = h^2$, but at section ②, the effective flow area is reduced by the wall boundary layers. Using the displacement thickness concept,

$$A_{2,eff} = (h - 2\delta_2^*)^2, \text{ so that } \left(\frac{A_1}{A_{2,eff}}\right)^2 = \left[\frac{h^2}{(h-2\delta_2^*)^2}\right]^2$$

Thus $$p_1 - p_2 = \frac{\rho V_1^2}{2}\left\{\left[\frac{h^2}{(h-2\delta_2^*)^2}\right]^2 - 1\right\}$$

$$= \frac{1}{2} \times 1.23\,\frac{kg}{m^3} \times (30)^2 \frac{m^2}{s^2}\left\{\left[\frac{(80)^2\,mm^2}{(80-2)^2\,mm^2}\right]^2 - 1\right\}\frac{N\cdot s^2}{kg\cdot m}$$

$p_1 - p_2 = 59.0\ N/m^2 \quad (59.0\ Pa)$ $\qquad p_1 - p_2$