Control Volume Forms of the Fundamental Laws

1. Conservation of Mass
2. Conservation of Linear Momentum
3. Conservation of Angular Momentum (moment of momentum)
4. Conservation of Energy

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Derivation of the Reynolds Transport Theorem

What: A formal mathematical expression which allows the time rate of change of an extensive property for a given quantity of mass, a system, to be expressed in terms of quantities related to a specific region of space, a control volume.

Why: All conservation law are written for system. We need to a way to express the time rate of change for a system in terms of a control volume. That is, since it is difficult to identify and follow the same mass of fluid, we need an Eularian description.

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Definitions

- System - A fixed collection of mass particles
- Control Volume – defined region in space
- Extensive Property (N) – property of the system that depends on mass ("Stuff").
  - Momentum, internal energy, entropy
- Intensive Property (η) – property of the system that is independent of mass
  - Temperature, velocity, specific energy

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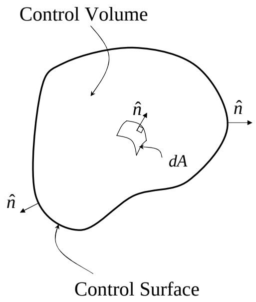
Basic Concepts



<u>Objective:</u>
Describe the rate at which an integral quantity associated with the system is changing as the flow passes into and out of the Control Volume.

Note: Fluid DOES NOT pass in and out of a system.

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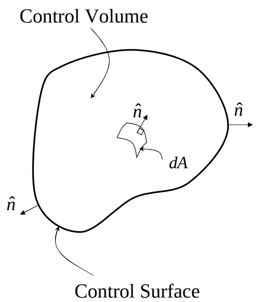
Basic Concepts



<u>Integral Properties:</u>

Mass flow rate:

$$\dot{m} = \int_A \rho \vec{V} \bullet \hat{n} dA$$

Mass:

$$m = \int_\forall \rho d\forall$$

Drag Force:

$$F_d = \int_A \vec{\tau} \bullet \hat{s} dA$$

Kinetic Energy:

$$KE = \int_\forall \rho \frac{V^2}{2} d\forall$$

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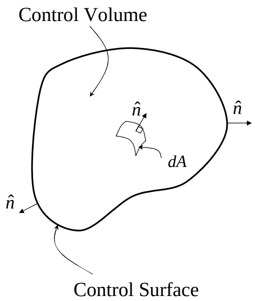
<u>Extensive/Intensive Properties</u>



$$N = \int_\forall \rho \eta d\forall$$

$$\eta = \frac{N}{m}$$

Momentum:

$$N = m\vec{V}$$

$$\eta = \frac{m\vec{V}}{m} = \vec{V}$$

Mass:

$$N = m$$

$$\eta = \frac{m}{m} = 1$$

<u>System</u>

$$N_{sys} = \int_{\forall_{sys}} \rho \eta d\forall$$

<u>Control Volume</u>

$$N_{C.V.} = \int_{C.V.} \rho \eta d\forall$$

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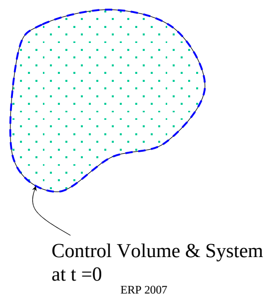
Reynolds Transport Theorem for a Deformable Control Volume



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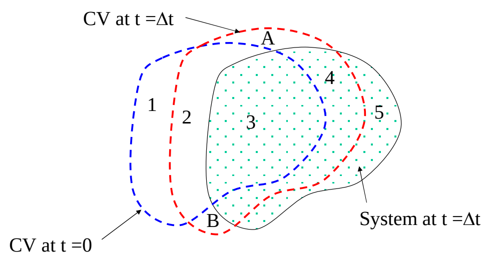
Reynolds Transport Theorem for a Deformable Control Volume



Region 6 = 2+A+B

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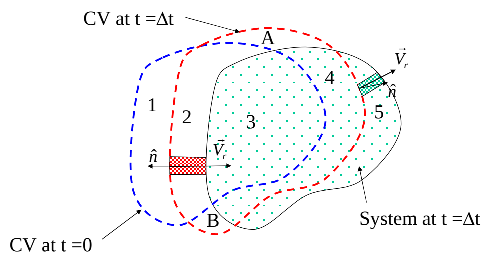
Reynolds Transport Theorem for a Deformable Control Volume



Region 6 = 2+A+B

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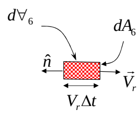
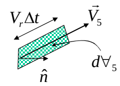
Finding the Size of the "Sweeping Volume"

Elemental volume from 2 - Incoming



$$d\forall_6 = -(V_r \bullet \hat{n})\Delta t dA_6$$

Elemental volume from 5 - Outgoing



$$d\forall_5 = (V_r \bullet \hat{n})\Delta t dA_5$$

$(V \bullet \hat{n})\Delta t$ = a length normal to the CS

$$N_6 = -\int_{A6.} \rho\eta(V_r \cdot \hat{n}) dA_6 \Delta t \qquad N_5 = \int_{A5.} \rho\eta(V_r \cdot \hat{n}) dA_5 \Delta t$$

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Reynolds Transport Theorem

$$\frac{DN_{sys}}{Dt} = \frac{d}{dt}\int_{CV} \eta\rho d\forall + \int_{CS} \eta\rho (V_r \cdot \hat{n}) dA$$

time rate of change of system of particles being followed

time rate of change term

flux term

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