Homework #1 – Calculus Review
ME5700/6700 Intermediate Fluid Dynamics
Due 9/5/2007

1. Use Gauss' Theorem ($\nabla \cdot \vec{B} = \lim_{V \to 0} \frac{1}{V} \int_S \vec{B} \cdot \hat{n} dS$) to show that for a cubical volume element in Cartesian coordinates that $\nabla \cdot \vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}$.

2. Stokes' Theorem
   a. Use Stokes' theorem ($(\nabla \times \vec{V}) \cdot \hat{n} = \lim_{S \to 0} \frac{1}{S} \oint \vec{V} \cdot d\vec{l}$, where S is the surface area) in Cartesian coordinates for an elemental rectangle to show that the z-component of the curl of the vector $\vec{V} = u_x \hat{i} + u_y \hat{j} + u_z \hat{k}$ is given by $\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}$.
   b. If $\vec{V} = \nabla \phi$, that is the vector $\vec{V}$ is given by the gradient of the scalar function ϕ, find the circulation of the vector $\vec{V}$ around some line L.

3. Give an example of an engineering problem (It does not have to be fluid mechanics related) that would require the use Leibniz's theorem where the velocity of at least one integration boundary is non-zero.

4. Transform the following equation $\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial^2 y}$, using $\eta = y / \sqrt{4\nu t}$. Take ν to be a constant.

5. Using summation notation where the subscripts, 1,2 and 3 represent the x, y and z directions respectively, write out:
   a. $\nabla \cdot \vec{V}$
   b. $(\vec{V} \cdot \nabla)\alpha$
   c. A force F that only acts in the y-direction.

6. Solve problems 1, 2, 3, 10 and 11 in Chapter 2 of Kundu page 47 and 48.