Homework #2 – Conservation Equation Concepts
ME5700/6700 Intermediate Fluid Dynamics
Due 9/21/2007

1. Consider fully developed, incompressible, laminar flow of a Newtonian fluid (with dynamic viscosity μ and density ρ) between two infinitely wide plates with the upper plate moving at a velocity, U and the bottom plate stationary.
   a. Solve for the velocity field. (Assume no horizontal pressure gradient and Neglect hydrostatic pressure)

   b. Write an expression for the stress and strain rate tensor.

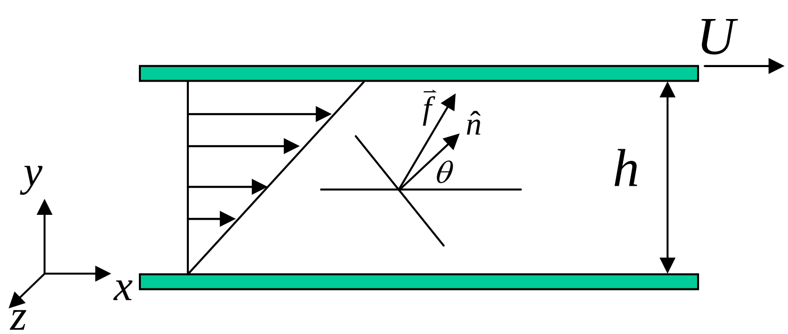
   c. Find the magnitude and direction of the force per unit area on an element whose outward normal points at $\theta = 50^o$ as shown in the figure below. Consider only the shear forces.

   d. Show that the viscous dissipation rate is $\mu U^2 / h^2$.

   e. Take a rectangular control volume for which the two horizontal surfaces coincide with the walls and the two vertical surfaces are perpendicular to the flow. Evaluate every term of the energy equation for this control volume and show that the balance is between the viscous dissipation and the work done in moving the upper surface.

   f. Consider, a similar problem but now with an axial pressure gradient applied to the flow, $\dfrac{\partial p}{\partial x} = a$, where a is constant. Solve for the new velocity field.

   g. Find the new magnitude and direction of force per unit area on an element whose outward normal points at $\theta = 30^o$. Consider only the shear forces.



2. A steady, two-dimensional, incompressible flow with constant properties in which the gravitational force is unimportant is described by the following velocity field (Note parts c and d are only required for students in the ME 6700 class):

$$\vec{V} = (ax + b)\hat{i} + (-ay + cx)\hat{j}$$

   a. Plot the velocity vector field (The easiest way to do this is using the "quiver" command in Matlab).
   b. Show that the velocity field satisfies the continuity equation
   c. Find an analytical expression for the pressure field
   d. Part c should give you a solution for the pressure to within an integration constant. Select a "reasonable" value for the integration constant and generate a pressure contour plot.