Homework #3 – Dimensional Analysis
ME5700/6700 Intermediate Fluid Dynamics
Due 10/5/2007 (5700 and 6700, please do all problems)

1. The spin associated with golf balls, cricket balls, baseballs, etc. is important to the flight trajectory of the ball. Given that the aerodynamic torque, *T* acting on a golf ball depends on flight speed, air density, air viscosity, ball diameter, spin rate and diameter of the dimples of the ball, by dimensional analysis, find the π–groups for this problem.

2. Consider the simplified momentum equation shown below for a steady, incompressible flow with constant viscosity and negligible body forces. The problem has a characteristic length scale *L* and velocity scale *U*.

$$\underbrace{\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)}_{A}=\underbrace{-\frac{\partial p}{\partial x}}_{B}+\underbrace{\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)}_{C}$$

Determine the proper non-dimensionalization of the pressure if (a) term $A \approx 0$ and if (b) term $C \approx 0$. Provide real world examples when this might be the case.

3. Kundu, Chapter 8, Problem 1
4. Kundu, Chapter 8, Problem 2

5. According to the one dimensional conduction model of a fin, the temperature distribution along the fin, $T(x)$, obeys the energy equation

$$\underbrace{kA\frac{d^2T}{dx^2}}_{\text{Longitudinal conduction}}-\underbrace{hP(T-T_o)}_{\text{Lateral convection}}+\underbrace{\dot{q}A}_{\text{Internal heat generation}}=0$$

Where *k, A, h, P* and $\dot{q}$ are the fin conductivity (W/m-K), fin cross-sectional area, fin-fluid heat transfer coefficient (W/m^2-K), the wetted perimeter of the fin that, as shown in the sketch below, is bathed by a fluid temperature T_o and is attached to a solid wall of temperature T_o. The heat generated by the fin is either absorbed by the fluid or the solid wall.

   a. As a system for scale analysis, select the fin section of length *x*, where *x* is measured away from the wall. Let T_∞ be the fin temperature sufficiently far from the wall. Show that if *x* is large enough, the longitudinal conduction term becomes negligible.
   b. Invoking the balance between lateral convection and internal heat generation, determine the fin temperature sufficiently far from the wall, T_∞.

c. Determine the fin section of length δ near the wall where the heat transfer is ruled by the balance between longitudinal conduction and internal heat generation.