CHAPTER 10

Exercise 10.1

We are required to solve $ff'' + f''' = 0$
subject to $f'(\infty) = 1$, $f(0) = 0$, and $f'(0) = 0$

To solve the problem by Runge-Kutta integration scheme, we need to reduce it to a set of 3 first order equations by defining

$$g(\eta) = f' \qquad \text{and} \qquad h(\eta) = f''$$

Then the problem reduces to

$$df/d\eta = g$$

$$dg/d\eta = h \tag{1}$$

$$dh/d\eta = -fh$$

subject to

$$f(0) = g(0) = 0 \tag{2}$$

$$g(\infty) = 1 \tag{3}$$

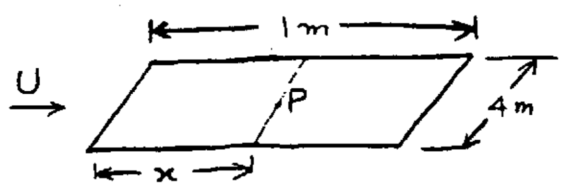
Procedure: Starting at $\eta = 0$, integrate (1) over successive steps $\Delta\eta$ by Runge-Kutta method, which is a standard computer library subroutine. Since we have initial conditions (at $\eta = 0$) for f and g only, we need to search for h(0) that will result in $g(\infty) = 1$. This can be done by trial and error. Because of the boundary-layer nature of the solution, condition (3) can be replaced by $g = 1$ at $\eta = 10$ (say). The details of the procedure, including the computer algorithm, is given by Carnahan, Luther and Wilkes (1969, page 407) *Applied Numerical Methods*, Wiley.

Exercise 10.2

$\nu = 2.29\times10^{-6}$ m^2/s

$\rho = 800$ kg/m^3

$U = 0.5$ m/s



$Re_L = UL/\nu = (0.5)(1)/2.29\times10^{-6} = 2.18\times10^5 < Re_{cr} = 10^6$

Therefore the flow is laminar everywhere.

At x = 0.5 m

$Re_x = Ux/\nu = 1.09\times10^5$. From equation (10.35): $\delta/x = 4.9/\sqrt{Re_x}$

$\therefore \quad \delta = 4.9x/\sqrt{Re_x} = 4.9(0.5)/\sqrt{1.09\times10^5} = 0.742$ cm

From (10.36): $\tau_0 = 0.332\rho U^2/\sqrt{Re_x} = 0.332(800)(0.5)^2/\sqrt{1.09\times10^5}$

$= 0.2\ N/m^2$

At x = 1 m

$\delta = 4.9L/\sqrt{Re_L} = 4.9(1)/\sqrt{2.18\times10^5} = 1.05$ cm

$\tau_0 = 0.332\rho U^2/\sqrt{Re_L} = 0.142\ N/m^2$

Total drag

From (10.38): $C_D = 1.33/\sqrt{Re_L} = 2.849\times10^{-3}$

$D = C_D(\frac{1}{2}\rho U^2)(bL) = (2.849\times10^{-3})(0.5)(800)(0.5)^2(4)(1) = 1.13$ N

Exercise 10.3

Given $\rho = 1.167\ kg/m^3$
$\nu = 1.5\times10^{-5}\ m^2/s$
$U = 6$ m/s
$u/U = 0.456$
$x = 15$ cm
$y, v = ?$

Calculate y

From Fig 10.5: $\eta = y\sqrt{U/\nu x} = 1.4$ when $u/U = 0.456$

$\therefore \quad y = 1.4\sqrt{\nu x/U} = 1.4\sqrt{(1.5\times10^{-5})(0.15)/6} = 8.57\times10^{-4} = 0.857$ mm

Calculate v

From Fig 10.6: $v\sqrt{x/\nu U} = 0.16$ when $\eta = 1.4$

$\therefore\ v = 0.16\sqrt{\nu U/x} = 0.16\sqrt{(1.5\times10^{-5})(6)/0.15} = 0.39\times10^{-2}$ m/s

Exercise 10.4

Given $\qquad u/U = \sin(\pi y/2\delta)$

Momentum integral equation is (10.42):

$$\frac{d}{dx}\int_0^{\delta}(U - u)u\,dy = \frac{\tau_o}{\rho}$$

$$\text{RHS} = \frac{\tau_o}{\rho} = \nu\left(\frac{\partial u}{\partial y}\right)_o = \nu\pi U/2\delta$$

$$\text{LHS} = \frac{d}{dx}\int_0^{\delta}\left(U - U\sin\frac{\pi y}{2\delta}\right)U\sin\frac{\pi y}{2\delta}\,dy = \frac{U^2}{2\pi}(4 - \pi)\frac{d\delta}{dx}$$

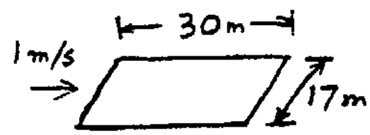
Momentum equation gives $\qquad \dfrac{U^2}{2\pi}(4 - \pi)\dfrac{d\delta}{dx} = \dfrac{\nu\pi U}{2\delta}$

Integration gives

$$\delta = \sqrt{\frac{2\pi^2}{4-\pi}}\sqrt{\frac{\nu x}{U}} = 4.795\sqrt{\frac{\nu x}{U}}$$

$$C_f = \frac{\tau_o}{\frac{1}{2}\rho U^2} = \frac{\nu\pi}{\delta U} = 0.655/\sqrt{Re_x}$$

Exercise 10.5



$Re_L = UL/\nu = (1)(30)/10^{-6} = 3\times10^{7} > Re_{cr} = 10^{6}$

From Fig 10.9: $\qquad C_D \equiv D/\frac{1}{2}\rho U^2 A = 0.00\cancel{5}$ 2.5

.0025 $\quad$ 2550

$\therefore\ D = \frac{1}{2}\rho U^2 A C_D = \frac{1}{2}(1000)(1)^2(30)(17)(0.005) = 1275$ N

Exercise 10.6

Fall velocity $\quad U = \sqrt{2gh} = \sqrt{2(9.81)(2.5)} = 7.0$ m/s

At steady state the drag on parachute equals the load, so that D = 80(9.81) N. Parachute cross sectional area is

$A = D/C_D\frac{1}{2}\rho U^2 = 80(9.81)/(2.3)(0.5)(1.167)(7)^2 = 11.93\ m^2 = \pi d^2/4$