Exercise 13.5

Given $w_{rms} = 1$ m/s

$T'_{rms} = 0.1^\circ C$

$r = 0.5$

$$Q = \rho C_p \overline{wT'} = \rho C_p (0.5 w_{rms} T'_{rms}) = (1.2)(1012)(0.5)(1)(0.1)$$
$$= 60.7 \text{ W/m}^2$$

Exercise 13.6

$$\text{Power} = mC_p \Delta T/\Delta t = (10)(4200)(1)/3600 = 11.67 \text{ W}$$

$$\text{Dissipation } \epsilon = \text{Power}/m = 11.67/10 = 1.667 \text{ m}^2/\text{s}^3$$

$$\therefore \eta = (\nu^3/\epsilon)^{1/4} = (10^{-18}/1.667)^{1/4} = 2.78\text{X}10^{-5} \text{ m} = 0.0278 \text{ mm}$$

Exercise 13.7

Given $D = 0.2$ m

$dp/dx = 8 \text{ N/m}^2$ per m

We know $dp/dx = 2\tau_o/R$. Thus

$$\tau_o = \frac{D}{4}\frac{dp}{dx} = (0.2)(8)/4 = 0.4 \text{ N/m}^2$$
$$u_* = \sqrt{\tau_o/\rho} = \sqrt{0.4/1000} = 0.02 \text{ m/s}$$

From Section 12 the viscous sublayer thickness is

$$\delta_\nu \simeq 5\nu/u_* = 5(10^{-6})/0.02 = 2.5\text{X}10^{-4} \text{ m} = 0.25 \text{ mm}$$

Exercise 13.8

Given $U = \frac{u_*}{k}\log y + C$

In the viscous sublayer the velocity profile is linear: $U = u_*^2 y/\nu$. At the edge of the viscous sublayer of thickness $\delta_\nu \simeq 10.7\nu/u$, the velocity is

$$U_\delta = u_*^2 \delta_\nu/\nu = \frac{u_*^2}{\nu}\left(\frac{10.7\nu}{u_*}\right) = 10.7u_*$$

Solutions to Homework #6 – Introduction to Turbulence
ME5700/6700 Intermediate Fluid Dynamics
Due 12/7/2007 (Anytime during the day)

Read Chapter 13 in Kundu and "The Turbulence Problem: An experimentalist's perspective," by Robert Ecke 2004.

1. Problems 6 and 7 in Kundu Chapter 13 (see posted solution).

2. In words describe the turbulent Energy Cascade Process
   -The energy cascade is process by which turbulent kinetic energy is transferred from the large scales of turbulence to progressively smaller scales of turbulence through a series of steps. This process is essentially inviscid, as the vortex stretching mechanism arises from the nonlinear equations of motion (See Kundu page 542 and Ecke pages 128-129 and Figure 4).

3. In Figure 3 in the Ecke paper please describe the difference between turbulent pipe flow and laminar pipe flow.
   - Turbulence enhances mixing in pipe flow and results in greater shear stresses at the wall (and greater pressure drops over a given distance) and a "flatter" velocity profile than the laminar case.

4. What is meant by "scale-independence" of turbulence?
   - Briefly, spatial or temporal signals look statistically the same under increasing magnification (see page 127, Ecke)

5. Describe how you might design an experiment to determine the "turbulent viscosity" in a flow field for a RANS model.
   - It is really necessary to simplify the problem, consider Kundu Ch. 13 pages 559-560. Also, a nice phenomenological description is given on page 553.
   - One of the simplest examples would be to consider simple 2D flow such as a plane mixing layer or turbulent boundary layer and use an x-array hot-wire anemometer to measure $\overline{u'v'}$ and profiles of the mean velocity. The eddy viscosity can then be determined through the relationship: $-\overline{u'v'} = \nu_T \frac{\partial \overline{u}}{\partial \overline{y}}$